Production-Inventory Systems: Modeling, Forecasting and Control

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Outline

• Dynamical Model of a Production-Inventory System
• Control Strategies:
  • IMC-PID and 2DoF Feedback-Only IMC
  • 3DoF Combined Feedback/Feedforward IMC
  • Model Predictive Control (MPC)
  • Improved MPC algorithm / Hybrid MPC
• Control-relevant Demand Modeling / Demand Forecasting
• Summary and Conclusions
Production-Inventory System

\[ y(t) = \frac{Ke^{-\theta s}}{s} u(s) - \frac{e^{-\theta_F s}}{s} d_F(s) - \frac{1}{s} d_U(s) \]

Integrating System with Delays
Semiconductor Manufacturing Supply Chain Management

**Fabrication/Sort**
- Nonlinear Throughput Time (~Weeks)
- Stochastic output

**Assembly/Test**
- Linear Throughput Time (~Days)
- Stochastic output

**Finish/Pack**
- Constant Throughput Time (~Shifts)
- Stochastic output

**Demand Factors**
- Stochastic demand
- Inaccurate forecasts
Global Warming/Climate Change

- From National Geographic Magazine
  (http://ngm.nationalgeographic.com/big-idea/05/carbon-bath)
Parental function $PF(t)$ is built up by providing an intervention $I(t)$ (frequency of home visits), that is potentially subject to delay, and is depleted by potentially multiple disturbances (adding up to $D(t)$).

\[ PF(t + 1) = PF(t) + K_I I(t - \theta) - D(t) \]

Internal Model Control (IMC) Design Procedure

• Step 1 (Nominal Performance): Obtain an $H_2$ (ISE)-optimal $q(s)$
  - An external input form is specified (e.g., step or ramp)
  - Closed-form solution for $q(s)$ is obtained
  - Resulting controller is stable and causal

• Step 2 (Robust Stability and Performance)
  - Augment the IMC controller from Step 1 with a filter, $f(s)$.
  - Proper choice and tuning of the filter ensures that:
    - the final controller $q(s)$ is proper.
    - the control system achieves stability and performance under uncertainty.
IMC-PID Tuning Rules

\[ p(s) = \frac{K e^{-\theta s}}{s} \]
\[ \tilde{p}(s) = \frac{K(-\frac{\theta}{2}s + 1)}{s\left(\frac{\theta}{2}s + 1\right)} \]
\[ q(s) = \frac{s}{K(\lambda s + 1)} \]

Representing the delay with a first-order Padé approximation and applying the IMC design procedure leads to the PID with filter controller.

\[ c(s) = K_c \left( 1 + \frac{1}{\tau I s} + \tau_D s \right) \frac{1}{(\tau_F s + 1)} \]
\[ K_c = \frac{3\theta + 4\lambda}{K(\theta^2 + 4\theta\lambda + 2\lambda^2)}, \quad \tau_I = \frac{3}{2}\theta + 2\lambda \]
\[ \tau_D = \frac{\theta^2 + 2\theta\lambda}{3\theta + 4\lambda}, \quad \tau_F = \frac{\theta\lambda^2}{\theta^2 + 4\theta\lambda + 2\lambda^2} \]

IMC-PID Controller Response

\[ K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \lambda = 5 \]
Two Degree-of-Freedom (2DoF) Feedback-Only IMC

\[ p(s) = \frac{K e^{-\theta s}}{s} \]

\[ \tilde{p}(s) = \frac{K e^{-\theta s}}{s} \]

No approximation is applied to the plant delay.

\[ q_r(s) = \frac{s}{K} \frac{1}{(\lambda_r s + 1)^{n_r}} \]

\[ q_d(s) = \frac{s(\theta s + 1)}{K} \frac{(n_d \lambda_d s + 1)}{(\lambda_d s + 1)^{n_d}} \]

2DoF Feedback-Only IMC

\[ K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \lambda_r = 1 \quad n_r = 2 \quad \lambda_d = 2 \quad n_d = 3 \]
3DoF Combined Feedback/Feedforward IMC Control

\[ p(s) = \frac{Ke^{-\theta s}}{s} \]
\[ \tilde{p}(s) = \frac{Ke^{-\theta s}}{s} \]
\[ p_d(s) = \frac{e^{-\theta_F s}}{s} \]
\[ \tilde{p}_d(s) = \frac{e^{-\theta_F s}}{s} \]
\[ q_r(s) = \frac{s}{K} \frac{1}{(\lambda_r s + 1)^{n_r}} \]
\[ q_d(s) = \frac{s(\theta s + 1)}{K} \frac{(n_d \lambda_d s + 1)}{(\lambda_d s + 1)^{n_d}} \]
\[ q_F(s) = \frac{e^{-(\theta_F - \theta_d - \theta)s}(n_F \lambda_F s + 1)}{K (\lambda_F s + 1)^{n_F}} , \quad \theta_F \geq \theta + \theta_d \]

3DoF Combined Feedback/Feedforward IMC Control

\[ K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \theta_F = 10 \]

\[ \lambda_r = 1 \quad n_r = 2 \quad \lambda_F = 1 \quad n_F = 3 \quad \lambda_d = 2 \quad n_d = 3 \]
Model Predictive Control (MPC)

\[
\begin{align*}
\min_{\Delta u(k|k) \ldots \Delta u(k+M-1|k)} & \sum_{\ell=1}^{P} Q_e(\ell)(\hat{y}(k+\ell|k) - r(k+\ell))^2 + \sum_{\ell=1}^{M} Q_{\Delta u}(\ell)(\Delta u(k+\ell - 1|k))^2 \\
\text{Keep Inventories at Planning Setpoints} & \quad \text{Penalize Changes in Factory Starts}
\end{align*}
\]
IMC/MPC Comparison

\[ \theta = 5, \ P = 20, \ M = 10, \ Q_e = 1, \ Q_{\Delta u} = 10 \]

- **Net Stock**
  - MPC
  - IMC

- **Factory Starts**
  - MPC
  - IMC

- **Customer Demand**
  - Actual Demand
  - \( \theta_F \) – day ahead Forecast

Time (Days)
Constrained MPC (with Stpt Anticipation)

Simulation under conditions of active constraints in net stock and factory starts.
Some Observations

- Feedback-only control strategies (even if multi-degree-of-freedom) are unsatisfactory (in general).

- Combined feedback-feedforward strategies that rely on the availability of a demand forecast signal are necessary for good, comprehensive control.

- Model predictive control can provide useful functionality (e.g., constraint handling, anticipation) but the traditional move suppression/single-degree-of-freedom formulation can be lacking.
Motivation for an Improved MPC Formulation

- Integrating dynamics (i.e., ramp responses and disturbances)

- Need to take advantage of anticipated future system inputs (i.e., forecasted demand)

- Multiple degrees-of-freedom (forecasted + unforecasted demand + inventory setpoint tracking) with ease of tuning

- Ability to incorporate problem-specific constraints and possibly hybrid dynamics

- Robustness in the presence of stochasticity and nonlinearity

Block Diagram for 3 DoF MPC Controller

- **MPC Controller**
  - **Forecasted Demand**
  - **Optimization**
  - **Filter II**
  - **Prediction and Estimation**
  - **Inventory Targets**
  - **Filter I**
  - **Actual Demand**
  - **Plant**
  - **Measurement Noise**
  - **Target**
  - **Error Projection**
  - **y**
  - **u**
Three Degree-of-Freedom (3-DoF) MPC Tuning

1. Filter I for inventory target setpoint tracking (Type I /asymptotically step signals)

\[ f_i(z) = \frac{(1 - \alpha_{Ii})z}{z - \alpha_{Ii}}, \quad i = 1, \ldots, n \]

2. Filter II for forecasted demand satisfaction (Type II /asymptotically ramp signals)

\[ f_j(z) = \frac{\left(1 - \alpha_{IIj} \right) + \frac{3}{5} \alpha_{IIj}}{1 - \alpha_{IIj} z^{-1}} - \frac{1}{5} \alpha_{IIj} z^{-1} - \frac{2}{5} \alpha_{IIj} z^{-2}, \quad j = 1, \ldots, n \]

Step-A1: $X(k|k - 1)$: one step ahead prediction using actual measured disturbance ($d$)

Step-A2: $X(k|k) = X(k|k - 1) + K_f(y(k) - CX(k|k - 1))$

Step-B1: $X_{flt}(k|k - 1)$: one step ahead prediction using filtered measured disturbance ($d_{flt}$)

Step-B2: $X_{flt}(k|k) = X_{flt}(k|k - 1) + K_f(y(k) - CX(k|k - 1))$

\[
K_f = [0 \ F_b \ F_a]^T
\]

\[
F_a = \text{diag}\{(f_a)_1, \cdots, (f_a)_{n_y}\}
\]

\[
F_b = \text{diag}\{(f_b)_1, \cdots, (f_b)_{n_y}\}
\]

\[
(f_b)_j = \frac{(f_a)_j^2}{1 + \alpha_j - \alpha_j (f_a)_j}, \quad 0 \leq (f_a)_j \leq 1, \quad 1 \leq j \leq n_y
\]

- $(f_a)_j$ is focused on each output $j$; constrained to $0 \leq (f_a)_j \leq 1$
- Speed of dist. rejection is proportional to the tuning parameter $(f_a)_j$
3-DoF MPC for Continuous Input

\[ f_a = 0.3; \alpha_r = 0.9; \alpha_d = 0.9 \]

\[ f_a = 1; \alpha_r = 0; \alpha_d = 0 \]

Independent controller adjustment without the need for move suppression!
Controller Model (includes hybrid dynamics)

Plant Model Mixed Logical Dynamical (MLD) Framework

\[
\begin{align*}
x(k+1) &= A x(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k) + B_d d(k) \\
y(k+1) &= C x(k+1) + d'(k+1) + \nu(k+1) \\
E_5 &\geq E_2 \delta(k) + E_3 z(k) - E_4 y(k) - E_1 u(k) + E_d d(k)
\end{align*}
\]

\[d': \text{ Unmeasured disturbance} \quad d : \text{ Measured disturbance}\]

Disturbance Model

\[
\begin{align*}
x_w(k+1) &= A_w x_w(k) + B_w w(k) \quad \text{Integrated white noise} \\
d'(k+1) &= C_w x_w(k+1) \\
A_w &= \text{diag}\{\alpha_1, \alpha_1, \cdots, \alpha_{n_y}\}, \quad B_w = C_w = I
\end{align*}
\]
MPC Objective Function

\[
\min_{\{u(k+i)_{i=0}^{m-1}, \delta(k+i)_{i=0}^{p-1}, z(k+i)_{i=0}^{p-1}\}} J \triangleq \sum_{i=1}^{p} \| (y(k+i) - y_r) \|_Q^2 + \sum_{i=0}^{m-1} \| (\Delta u(k+i)) \|_{Q_{\Delta u}}^2 \\
+ \sum_{i=0}^{m-1} \| (u(k+i) - u_r) \|_{Q_u}^2 + \sum_{i=0}^{p-1} \| (\delta(k+i) - \delta_r) \|_{Q_d}^2 + \sum_{i=0}^{p-1} \| (z(k+i) - z_r) \|_{Q_z}^2
\]

Subject to

\[
E_5 \geq E_2 \delta(k+i) + E_3 z(k+i) - E_4 y(k+i) - E_1 u(k) + E_d d(k+i), \quad 0 \leq i \leq p - 1 \\
y_{\min} \leq y(k+i) \leq y_{\max}, \quad 1 \leq i \leq p \\
u_{\min} \leq u(k+i) \leq u_{\max}, \quad 0 \leq i \leq m - 1 \\
\Delta u_{\min} \leq \Delta u(k+i) \leq \Delta u_{\max}, \quad 0 \leq i \leq m - 1
\]
Hybrid 3 DoF Model Predictive Control, Production-Inventory System

\[ u(k) \in \{0, 33.33, 66.66, 100\} \]

\[ y(k + 1) = y(k) + Ku(k - (\theta - 1)) - d(k) \]

\[ d(k) = \begin{cases} df(k) & \text{forecasted} \\ du(k) & \text{unforecasted} \end{cases} \]
Hybrid vs Continuous 3 DoF MPC
Production-Inventory System

**Continuous** $u(t)$

- $y(k)$
- $u(k)$
- $d(k)$

**Discrete-level** $u(t)$

- $u(k) \in \{0, 33.33, 66.66, 100\}$

Solution involves solving a *Mixed Integer Quadratic Program* (MIQP) to address continuous error but discrete-level inputs (i.e., a hybrid problem).
Production-Inventory System in the Presence of Forecast Error

\[ d(t) = d_F(t - \theta_F) + d_U(t) \]

Integrating System with Delays
The closed-loop system response to a unit pulse in forecast error provides a basis for understanding modeling requirements for control-relevant demand models.

The effect of forecast error on closed-loop performance is most significant in an intermediate frequency range.
Response to Forecast Error (MPC, changing move suppression)

Inventory response to forecast error

Starts response to forecast error

\( Q_{u} = 1.0 \)
True demand is defined by a demand transfer function $p_d(z)$ and a stochastic component $H(z)a(t)$.

$$d(t) = p_d(z)u_d(t) + H(z)a(t)$$

The estimated demand is defined by $\tilde{p}_d(z)$ and a noise model $\tilde{p}_e(z)$.

$$d(t) = \tilde{p}_d(z)u_d(t) + \tilde{p}_e e(t)$$

The control-relevant estimation step consists of minimizing the one-step-ahead prediction error, where $L(z)$ is the prefilter.

$$\min_{\tilde{p}_d,\tilde{p}_e} V = \min_{\tilde{p}_d,\tilde{p}_e} \frac{1}{N} \sum_{t=1}^{N} [L(z)e(t)]^2 = \min_{\tilde{p}_d,\tilde{p}_e} \frac{1}{N} \sum_{t=1}^{N} e_L^2(t)$$

Parseval’s theorem allows for frequency domain analysis of the problem.

$$\lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} e_L^2(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \frac{L(e^{j\omega})}{\tilde{p}_e(e^{j\omega})} \right)^2 \left( |p_d(e^{j\omega}) - \tilde{p}_d(e^{j\omega})|^2 \Phi_{u_d}(\omega) + |H(e^{j\omega})|^2 \Phi_a(\omega) \right) d\omega$$
Multi-Objective Formulation

It is desirable to minimize a weighted combination of inventory and factory starts variance.

\[
\min_{\tilde{p}_d, \tilde{p}_e} \left[ \sum_{t=0}^{\infty} (1 - \gamma) e_c^2(t) + \lambda \sum_{t=0}^{\infty} \gamma \Delta u^2(t) \right]
\]

The control-relevant prefilter then takes the following form.

\[
\frac{|L(e^{j\omega})|^2}{|\tilde{p}_e(e^{j\omega})|^2} \Phi_{e_F}(\omega) = (1 - \gamma)|L_{e_c}(e^{j\omega})|^2 \Phi_{e_F}(\omega) + \gamma \lambda |L_{\Delta u}(e^{j\omega})|^2 \Phi_{e_F}(\omega)
\]

By assuming an output error model structure, \( L(z) \) can be reduced to the following form.

\[
|L(e^{j\omega})|^2 = (1 - \gamma)|L_{e_c}(e^{j\omega})|^2 + \gamma \lambda |L_{\Delta u}(e^{j\omega})|^2
\]

A curve fitting procedure is then used to obtain an Infinite Impulse Response filter that matches the amplitude ratio of the control-relevant prefilter.
Multi-Objective Formulation (Cont.)

\[ |L(e^{j\omega})|^2 = (1 - \gamma)|L_e(e^{j\omega})|^2 + \gamma \lambda |L_\Delta u(e^{j\omega})|^2 \]

\( \gamma = 0 \) : Inventory Variance Optimal
\( \gamma = 1 \) : Starts Change Variance Optimal
\( \gamma \) : Weighted Combination
Final Observations

- Production-inventory systems are iconic dynamical systems that describe interesting problems in the process industries (and beyond).

- Combined feedback-feedforward strategies relying on demand forecast signals are necessary to adequately control these systems. Improved formulations of MPC can be developed in this regard.

- Demand modeling is a problem of significant importance in production-inventory systems; analysis of closed-loop decision policies show that these are most responsive to forecast error in an intermediate frequency bandwidth.

- Prefiltering can be used to apply the proper emphasis in control-relevant demand modeling.

- Multivariable extensions exist for both the control and demand modeling / demand forecasting segments of this presentation.
Primary References


Additional references in [http://csel.asu.edu/SCMpapers](http://csel.asu.edu/SCMpapers)
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