Research on Economic Performance Assessment and Diagnosis of Industrial MPC

Hongye Su

National Key Laboratory of Industrial Control Technology
Institute of Cyber-Systems and Control
Zhejiang University, China
Petrochemical Industry

1. Pillar industry in the world
   - The first major pillar industry in the world
   - $14.9 trillion gross output in the world (2013)
   - China as number one

2. Big energy producer, Big energy user
   - 15% of total energy consumption
   - 15%~20% above the average energy consumption level

MPC: Enabling Technology of Saving Energy and Increasing Profit
Outline

1. Why CPA
2. CPA of PID Loop
3. Economic PA of Industrial MPC
4. On-line EPI of Industrial MPC
5. MPM Detection of MPC with Mutual Information

MPC: Model Predictive Control
Control System: Big Investment

1. Typical Control Loop Investment: $25,000 (ABB Company)
   - Hardware: Including valve, sensor, controller etc.
   - Software: Control algorithm, SCADA system etc.

2. Typical Petrochemical Process: $10^2 \sim 10^3$ Loops

Improperly Working Control System

WHAT A WASTE OF MONEY
However:

1. Fewer and fewer adequately educated control engineer

2. Average control engineer responsible $> 100$ loops

Short of Maintenance

Control Performance Reality: Not Good
Outline

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MPC: Model Predictive Control
PID Regulatory Performance

Measured by Loop Output Variance

\[ n = [n_0, n_1, \ldots, n_{m-1}]^T \]

Disturbance Impulse Response

\[ \sigma_y^2 \approx n^T(I + SC)^{-T}(I + SC)^{-1}n \]

- Nonparameter-Model Based
- Applied to any order process

Good or Bad?
Good or Bad?

Find the benchmark:
Minimum Output Variance

$$\min_C n^T(I + SC)^{-T}(I + SC)^{-1} n$$

Nonconvex!
CPA of PID

Good or Bad?

\[ \min_C \mathbf{n}^T (I + SC)^{-T} (I + SC)^{-1} \mathbf{n} \]

Nonconvex!

Nonconvex Constraints

\[ \begin{align*}
\min_{A, x, V, z} & \quad z \\
\text{s.t.} & \quad A = H((GG^T) \odot (QVQ^T))H^T \\
& \quad \begin{bmatrix} A & \mathbf{n} \\ \mathbf{n}^T & z \end{bmatrix} \succeq 0, \\
& \quad V \succeq 0, \quad \begin{bmatrix} V & \mathbf{x} \\ \mathbf{x}^T & 1 \end{bmatrix} \succeq 0 \\
& \quad \text{trace}(V) \leq \mathbf{x}^T \mathbf{x}.
\end{align*} \]
Good or Bad?

Nonconvex!

\[ \min_C n^T(I + SC)^{-T}(I + SC)^{-1} n \]

Lagrange method & Fixed-point Alg.

Schur Complement

Nonconvex Constraints
 CPA of PID  

**Good or Bad?**

\[
\min_C n^T(I + SC)^{-T}(I + SC)^{-1}n
\]

**Nonconvex!**

\[
x^{(k)} = \arg\min_{A, x, V, z} \quad z + \lambda(\text{trace}(V) - (2x^{(k-1)^T}x - x^{(k-1)^T}x^{(k-1)}))
\]

s.t.  \[z \geq 0, \quad z \leq [\sigma_y^{2(k-1)}],\]

\[A = H((GG^T) \odot (QVQ^T))H^T,\]

\[
\begin{bmatrix}
A & n \\
 n^T & z
\end{bmatrix} \succeq 0, \quad \begin{bmatrix}
V & x \\
 x^T & 1
\end{bmatrix} \succeq 0,
\]

\[V \succeq 0, \quad [V]_{1,1} = 1, \quad x = [V]_1,\]

\[
\min_{A, x, V, z} \quad z
\]

s.t.  \[z \geq 0, \quad A = H((GG^T) \odot (QVQ^T))H^T\]

\[
\begin{bmatrix}
A & n \\
 n^T & z
\end{bmatrix} \succeq 0,
\]

\[V \succeq 0, \quad \begin{bmatrix}
V & x \\
 x^T & 1
\end{bmatrix} \succeq 0
\]

\[\text{trace}(V) \preceq x^T x.\]

**Lagrange method & Fixed-point Alg.**

**Schur Complement**

**Nonconvex Constraints**
CPA of PID

Good or Bad?

**Nonconvex!**

\[
\min_c n^T(I + SC)^{-T}(I + SC)^{-1}n
\]

**Successive Convex Problem**

\[
x^{(k)} = \arg\min_{\Lambda, x, V, z} \quad z + \lambda \text{trace}(V) - (2x^{(k-1)T}x - x^{(k-1)T}x^{(k-1)})
\]

\[
V \succeq 0, \quad [V]_{1,1} = 1, \quad x = [V]_1,
\]

**Lagrange method & Fixed-point Alg.**

\[
\min_{\Lambda, x, V, z} \quad z
\]

\[
\text{s.t.} \quad z \geq 0, \quad A = H((GG^T) \odot (QVQ^T))H^T
\]

\[
\begin{bmatrix} A \\ n^T \\ z \end{bmatrix} \succeq 0,
\]

\[
V \succeq 0, \quad \begin{bmatrix} V & x \\ x^T & 1 \end{bmatrix} \succeq 0
\]

\[
\text{trace}(V) \leq x^T x.
\]

**Nonconvex Constraints**
CPA of PID

Succesive Convex Problem

Algorithm 1. The proposed iterative convex programming approach

Initialization: (i) Set the numerical tolerance $\epsilon$ and the maximum number of iterations $K^{(\text{max})}$; (ii) Compute an initial point $x^{(0)}$ according to Section 4.2; (iii) Set the iteration number $k = 1$.
Repeat:
   Step 1: Solve the convex problem (19) to obtain the solution $x^{(k)}$.
   Step 2: Compute the output variance $[\sigma_y^2]^{(k)}$ with the obtained PID parameters $[c_1^{(k)}, c_2^{(k)}, c_3^{(k)}]$, and record the obtained results.
   Step 3: Update the iteration number $k = k + 1$.
Until: $|f(x^{(k)}) - f(x^{(k-1)})| < \epsilon$ or $k > K^{(\text{max})}$.

Readily Solved!

CPA: Compare current loop output variance with the benchmark
## CPA of PID

### Test on 100 typical single-loop

<table>
<thead>
<tr>
<th>Case</th>
<th>BKRs</th>
<th>$\sigma^2$ (Algorithm 1)</th>
<th>$[c_1, c_2, c_3]$ (Algorithm 1)</th>
<th>$\lambda$</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0728</td>
<td>3.0728</td>
<td>[2.8407, -4.4056, 1.7485]</td>
<td>1</td>
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<td>[1.9556, -3.6286, 1.6746]</td>
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<td>3</td>
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<td>3.0442</td>
<td>[0.6315, -1.2380, 0.6065]</td>
<td>$10^3$</td>
<td>4.3</td>
</tr>
<tr>
<td>4</td>
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<td>3.4081</td>
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<td>10</td>
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<tr>
<td>5</td>
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<td>13.8077</td>
<td>[0.7253, -1.2081, 0.5190]</td>
<td>$10^3$</td>
<td>466.8</td>
</tr>
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<td>6</td>
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<td>87.7380</td>
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<td>$10^4$</td>
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<tr>
<td>7</td>
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<td>0.4247</td>
<td>[8.0823, -13.1663, 5.5814]</td>
<td>1</td>
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<tr>
<td>9</td>
<td>0.4268</td>
<td>0.4268</td>
<td>[8.2316, -13.7790, 5.9699]</td>
<td>1</td>
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</tr>
<tr>
<td>10</td>
<td>0.0024</td>
<td>0.0024</td>
<td>[1.2867, -8.8138, 3.1626]</td>
<td>$10^{-2}$</td>
<td>513.8</td>
</tr>
</tbody>
</table>

Can obtain the benchmark accurately & quickly
Outline

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MPC: Model Predictive Control
MPC Performance Assessment (PA)

More Important & Challenging!

1. Widely applied petrochemical industry
   - > 50 new projects/year in China
   - > 400 in service in China

2. Control objective of MPC is highly related to economic profit of plant

3. Performance may degrade quickly
   - Typical 6 months good-performance-period after commissioning

DOC PA based on LQG

Given $E\{u^2\} \leq \alpha$, what is minimum of $E\{y^2\}$?

Varying $\lambda$

Solving the LQG problem

Obtain the MPC Performance Limit Curve

$\Phi = E\left[\|y - y^s\|_Q\right] + \lambda E\left[\|u - u^s\|_R\right]$

s.t.

Process Dynamic Model

$U_{i,\text{min}} \leq u_i \leq U_{i,\text{max}}$

$Y_{j,\text{min}} \leq y_j \leq Y_{j,\text{max}}$

$\mathbb{E} \{u^2\} \leq \alpha$, what is minimum of $\mathbb{E} \{y^2\}$?

Varying $\lambda$

Solving the LQG problem

Obtain the MPC Performance Limit Curve

$\Phi = E\left[\|y - y^s\|_Q\right] + \lambda E\left[\|u - u^s\|_R\right]$

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Process Dynamic Model

$U_{i,\text{min}} \leq u_i \leq U_{i,\text{max}}$

$Y_{j,\text{min}} \leq y_j \leq Y_{j,\text{max}}$
Economic PA of Double-Layer Industrial MPC

Double-Layer Structure of Industrial MPC

- Local equipments economic optimization
- Double-layer MPC
- PID basic control loop

Local optimization (hour)

Steady State Optimization (minute)

Dynamic optimal control (minute)

Basic loop dynamic control (second)

What is the best coordination of the two layers?

Economic Performance Assessment


Economic PA of Double-Layer Industrial MPC

\[
\begin{align*}
\max_{y_j^s, u_i^s, \sigma_{y_j}, \sigma_{u_i}} & \quad \Delta J = \sum_{j=1}^{p} C_{y}^{(j)} y_j^s - \sum_{i=1}^{m} C_{u}^{(i)} u_i^s \\
\text{s.t.} & \quad y_j^s = \sum_{i=1}^{m} k_{ij} u_i^s \\
& \quad \Delta u_i^s = u_i^s - u_i^{s0} \\
& \quad \Delta y_j^s = y_j^s - y_j^{s0} \\
& \quad Y_{j,\min} + z_{\alpha_j/2} \sigma_{y_j} \leq y_j^s \leq Y_{j,\max} - z_{\alpha_j/2} \sigma_{y_j} \\
& \quad U_{i,\min} + z_{\alpha_i/2} \sigma_{u_i} \leq u_i^s \leq U_{i,\max} - z_{\alpha_i/2} \sigma_{u_i} \\
& \quad \sigma_{y} \geq 0 \\
& \quad \sigma_{u} \geq 0 \\
& \quad \sigma_{y} = f(\sigma_{u}) \\
\end{align*}
\]

\[\Phi = E \left[ \| y - y^s \|_Q \right] + \lambda E \left[ \| u - u^s \|_R \right] \]

s.t.

Process Dynamic Model

\[U_{i,\min} \leq u_i \leq U_{i,\max}\]

\[Y_{j,\min} \leq y_j \leq Y_{j,\max}\]

I/O Variances determined in DOC

Steady State Optimization (SSO)

Dynamic Optimal Control (DOC)
Economic PA of Double-Layer Industrial MPC

\[
\max_{y_j^s, u_i^s, \sigma_y, \sigma_{u_i}} \Delta J = \sum_{j=1}^{p} C_y^{(j)} y_j^s - \sum_{i=1}^{m} C_u^{(i)} u_i^s
\]

s.t. \[
y_j^s = \sum_{i=1}^{m} k_{ij} u_i^s
\]

\[
\Delta u_i^s = u_i^s - u_i^{s0}
\]

\[
\Delta y_j^s = y_j^s - y_j^{s0}
\]

\[
Y_j, \min + z_{\alpha_j/2} \sigma_{y_j} \leq y_j^s \leq Y_j, \max - z_{\alpha_j/2} \sigma_{y_j}
\]

\[
U_i, \min + z_{\alpha_i/2} \sigma_{u_i} \leq u_i^s \leq U_i, \max - z_{\alpha_i/2} \sigma_{u_i}
\]

\[
\sigma_Y \geq 0
\]

\[
\sigma_U \geq 0
\]

\[
\sigma_Y = f(\sigma_U)
\]

Best I/O Variances Coordination based on LQG Benchmark

SSO

DOC
Economic Assessment Indexes

\[ \eta_E = \frac{\Delta J_E}{\Delta J_I} \leq 1 \]

**\( \Delta J_E \): Obtained economic performance**

**\( \Delta J_I \): Ideal economic performance**
Economic PA of Double-Layer Industrial MPC Application on Delayed Coking Furnace Control:

\[ \text{Max } \eta_e = 100 - [(c_1 + c_2 \cdot \theta_{air}) \cdot (y^s + c_3 \cdot (y^s)^2) - c_4] - \beta \]

\( \eta_e \) : Furnace Thermal Efficiency
\( y \) : Furnace Outlet O\(_2\) Concentration
Economic PA of Double-Layer Industrial MPC

Delayed Coking Furnace Control

LQG Benchmark Curve
Economic PA of Double-Layer Industrial MPC

Delayed Coking Furnace Control

Furnace Output O$_2$: 4.6% → 3.5%
Thermal Efficiency: 86.5% → 87.1%
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MPC: Model Predictive Control
On-line Performance Improvement of Double-Layer Industrial MPC

1. Requires an accurate process model
2. Computationally demanding

Off-line Performance Assessment

On-line Performance Improvement of Double-Layer Industrial MPC

Iterative Learning Control (ILC)
- Data-Driven
- Model-Free

On-line Economic Performance Improvement (EPI)

On-line Performance Improvement of Double-Layer Industrial MPC

\[
\max_{y_j^s, u_i^s, \sigma_y, \sigma_u} J = \sum_{j=1}^{p} C_y^{(j)} y_j^s - \sum_{i=1}^{m} C_u^{(i)} u_i^s \\
\text{s.t. } y_j^s = \sum_{i=1}^{m} k_{ij} u_i^s \\
\Delta u_i^s = u_i^s - u_i^{s0}
\]

Find Active Constraints

\[
y_{j, \min} + z_{\alpha_j/2} \sigma_{y_j} \leq y_j^s \leq y_{j, \max} - z_{\alpha_j/2} \sigma_{y_j}
\]

\[
U_{i, \min} + z_{\alpha_i/2} \sigma_{u_i} \leq u_i^s \leq U_{i, \max} - z_{\alpha_i/2} \sigma_{u_i}
\]

\[
\sigma_y \geq 0 \\
\sigma_u \geq 0
\]

\[
\sigma_y = f(\sigma_u)
\]

SSO

I/O variance from the operation data
On-line Performance Improvement of Double-Layer Industrial MPC

\[
\max_{y^s, u^i, \sigma_{y_j}, \sigma_{u_i}} J = \sum_{j=1}^{p} C^{(j)} y_j^s - \sum_{i=1}^{m} C^{(i)} u_i^s \\
\text{s.t. } y_j^s = \sum_{i=1}^{m} k_{ij} u_i^s \\
\Delta u_i^s = u_i^s - u_i^{s0}
\]

Find Active Constraints

\[
Y_{j,\min} + z_{\alpha_j/2} \sigma_{y_j} \leq y_j^s \leq Y_{j,\max} - z_{\alpha_j/2} \sigma_{y_j} \\
U_{i,\min} + z_{\alpha_i/2} \sigma_{u_i} \leq u_i^s \leq U_{i,\max} - z_{\alpha_i/2} \sigma_{u_i}
\]

\[
\sigma_Y \geq 0 \\
\sigma_U \geq 0 \\
\sigma_Y = f(\sigma_U)
\]

Sensitivity Analysis

ILC-based DOC Weights Retuning

\[
\Phi = E \left[ \| y - y^s \|_Q \right] + \lambda E \left[ \| u - u^s \|_R \right]
\]

s.t.

Process Dynamic Model

\[
U_{i,\min} \leq u_i \leq U_{i,\max} \\
Y_{j,\min} \leq y_j \leq Y_{j,\max}
\]
On-line Performance Improvement of Double-Layer Industrial MPC

\[
\max_{y^s_j, u^s_i, \sigma_{y^s_j}, \sigma_{u^s_i}} J = \sum_{j=1}^{p} C_{y}^{(j)} y_j^s - \sum_{i=1}^{m} C_{u}^{(i)} u_i^s \\
\text{s.t. } y_j^s = \sum_{i=1}^{m} k_{ij} u_i^s \\
\Delta u_i^s = u_i^s - u_i^{s0} \\
\Delta y_j^s = y_j^s - y_j^{s0}
\]

Active Constraints Relaxed

\[
\sigma_y \geq 0 \\
\sigma_u \geq 0 \\
\sigma_y = f(\sigma_u)
\]

SSO

I/O Variances Re-distributed
On-line Performance Improvement of Double-Layer Industrial MPC

Improved Economic Performance

\[ s.t. \quad y_j = \sum_{i=1}^{m} k_{ij} u_i \]

\[ \Delta u_i^s = u_i^s - u_i^{s0} \]

\[ \Delta y_j^s = y_j^s - y_j^{s0} \]

Active Constraints Relaxed

\[ \sigma_y \geq 0 \]

\[ \sigma_u \geq 0 \]

\[ \sigma_y = f(\sigma_u) \]

SSO

I/O Variances Re-distributed
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MPC: Model Predictive Control
1. Model is the core of MPC
   - MPC heavily relies on an accurate model to predict the process behavior

2. Model Plant Mismatch (MPM) is the No.1 root cause of poor MPC control performance

Mutual Information: $I(X; Y)$

- Given two random variables $X, Y$

$$I(X; Y) = H(X) + H(Y) - H(X, Y)$$

$$H(X) = -\int p(x) \log(p(x)) \, dx$$

$$H(Y) = -\int p(y) \log(p(y)) \, dy$$

$$H(X, Y) = -\int p(x, y) \log(p(x, y)) \, dx \, dy$$

$MI$ quantify the information shared by $X$ and $Y$

$I(X; Y) = 0$ iff $X$ and $Y$ are independent
MPM detection using MI

No MPM: \( e = v \) independent of \( u_d \)

\[
I(e; u_d) = 0
\]

MPM: \( I(e; u_d) \neq 0 \)
MI Estimation

- MI Estimation
  - K-nearest neighbor approach

\[
\hat{I}(X; Y) = \psi(k) - \frac{1}{k} - \frac{1}{N} \sum_{i=1}^{N} \left[ \psi(n_x(i)) + \psi(n_y(i)) \right] + \psi(N)
\]

- MI Statistic Confidence Limit
  - Surrogate data approach
    - iAAFT: iterative amplitude adjusted Fourier Transform
MPM Localization using MI

MIMO System

\[
\begin{bmatrix}
    u_{d_1} & u_{d_2} & \cdots & u_{d_n} \\
    , & , & \cdots & , \\
    u_{c_1} & u_{c_2} & \cdots & u_{c_n} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \Delta g_{11} & \Delta g_{12} & \cdots & \Delta g_{1n} \\
    0 & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    \Delta g_{m1} & \Delta g_{m2} & \cdots & \Delta g_{mn} \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_m
\end{bmatrix}
\]

If \( I(e_i; ud_j) \neq 0 \), then the \( j \)th column of \( \Delta G(:, j) \neq 0 \),
Polypropylene: General Purpose Plastic

Double-loop liquid propylene polymerization plant of SINOPEC Co. Ltd. (Zhenghai)
# MPC for R201, R202

## Table 1. MVs and CVs of the MPC for Tubular Reactors

<table>
<thead>
<tr>
<th>variable</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVs</td>
<td></td>
</tr>
<tr>
<td>MV1</td>
<td>flow of hydrogen</td>
</tr>
<tr>
<td>MV2</td>
<td>flow of propylene monomer</td>
</tr>
<tr>
<td>CVs</td>
<td></td>
</tr>
<tr>
<td>CV1</td>
<td>concentration of hydrogen</td>
</tr>
<tr>
<td>CV2</td>
<td>density of slurry</td>
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</table>
MPC Performance

Slurry density: Important quality Index

Control results of MPC at early commissioning stage
MPC Performance
Slurry density: Important quality Index

Control results of MPC after commissioning for 7 months
MPM Detection of R201

MPM exist in the channels of

$\textbf{MV}_1 \rightarrow e_1$ and $\textbf{MV}_2 \rightarrow e_1$
MPC Performance after Maintenance

Control results of MPC after model re-identification
Thank You!