Plug-and-Play Control and Optimization in Microgrids

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Paradigm shifts in the operation of power networks

Traditional **top to bottom** operation:
- generate/transmit/distribute power
- hierarchical control & operation

Smart & green **power to the people**:
- high renewable penetration
- distributed generation & deregulation
- demand response & load control

Microgrids

**Structure**
- low-voltage distribution networks
- grid-connected or islanded
- autonomously managed

**Applications**
- hospitals, military, campuses, large vehicles, & isolated communities

**Benefits**
- naturally distributed for renewables
- flexible, efficient, & reliable

**Operational challenges**
- volatile dynamics & low inertia
- plug’n’play & no central authority

Conventional control architecture from bulk power ntwks

1. **Primary control** (fast)
   - Goal: stabilization & load sharing
   - Strategy: decentralized

2. **Secondary control** (slower)
   - Goal: maintain operating point
   - Strategy: centralized

3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast

⇒ break vertical & horizontal hierarchy
A preview – plug-and-play control and optimization
flat hierarchy, distributed, no time-scale separations, & model-free . . .

### Outline

**Introduction**

**Primary Control**

**Tertiary Control**

**Secondary Control**

P-n-P Experiments

Conclusions

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Modeling: a microgrid is a circuit

- **synchronous ( & acyclic) AC circuit** with harmonic waveforms $E_i e^{i(\theta_i + \omega t)}$

- **ZIP loads**: constant impedance, current, & power $P_i^* + iQ_i^*$ *(today)*

- **coupling** via Kirchhoff & Ohm

- purely inductive lines $G/B \approx 0$ *(can be relaxed to $G/B = \text{const}$.)*

- decoupling: $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ *(near operating point)*

  - active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
  - reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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  - trigonometric active power flow: $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$
  - polynomial reactive power flow: $Q_i(E) = -\sum_j B_{ij} E_i E_j$ *(not today)*
Modeling: sources interfaced with inverters
(all results also apply to synchronous machines & frequency-dependent loads)

Power inverters are . . .
• interfaces between
  ◦ the AC microgrid and
  ◦ DC & variable AC sources
• controllable (voltage) sources
[Zhong & Hornik, '12]

\[ \omega_{\text{sync}} \]

Decentralized primary control of active power

Inverters are controlled to emulate the physics of synchronous generators.
[Chandorkar et. al. '93]

Intuition: Recall...

\[
P_i(\theta) = \sum_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j)
\]

\[
P_i \dot{\theta}_i \rightarrow \text{droop control: } \frac{1}{D_i} \left( P_i^* - P_i(\theta) \right)
\]

Putting the pieces together...
differential-algebraic closed loop

network physics

load power balance: \( P_i^* = \sum_j B_{ij} \sin(\theta_i - \theta_j) \)
source injections: \( P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j) \)

\[
\dot{\theta}_i = \frac{1}{D_i} \left( P_i^* - P_i(\theta) \right)
\]

\[
\text{loads: } 0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)
\]

\[
\text{sources: } D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)
\]
Closed-loop stability under droop control

Theorem: stability of droop control  
[J. Simpson-Porco, FD, & F. Bullo, ’12]

∃ unique & exp. stable frequency sync ⇐⇒ active power flow is feasible

Main proof ideas and some further results:
• synchronization frequency:  
  \[ \omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{inverters}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{inverters}} D_i} \]  
  (∝ power balance)

• steady-state power injections:  
  \[ P_i = \begin{cases} P_i^* & \text{for loads} \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & \text{for inverters} \end{cases} \]  
  (depend on \( D_i \) & \( P_i^* \))

• unique steady-state branch flows:  
  \[ \xi_{ij} = B_{ij} \sin(\theta_i^* - \theta_j^*) \Rightarrow B_{ij} \geq \xi_{ij} \]  
  (\( P_i \mapsto \xi_{ij} \))

Objective I: decentralized proportional load sharing

1) Inverters have injection constraints:  
   \( P_i(\theta) \in [0, \bar{P}_i] \)

2) Load must be serviceable:  
   \[ 0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{inverters}} \bar{P}_j \]

3) Fairness: load should be shared proportionally:  
   \( P_i(\theta)/\bar{P}_i = P_j(\theta)/\bar{P}_j \)

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Theorem: fair proportional load sharing  
[J. Simpson-Porco, FD, & F. Bullo, ’12]

Let the droop coefficients be selected proportionally:

\[ D_i/\bar{P}_i = D_j/\bar{P}_j \quad \& \quad P_i^*/\bar{P}_i = P_j^*/\bar{P}_j \]

The the following statements hold:

(i) Proportional load sharing:  
   \( P_i(\theta)/\bar{P}_i = P_j(\theta)/\bar{P}_j \)

(ii) Constraints met:  
   \[ 0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{inverters}} \bar{P}_j \ \Leftrightarrow \ P_i(\theta) \in [0, \bar{P}_i] \]
Objective I: fair proportional load sharing
proportional load sharing is not always the right objective

Objective II: optimal economic dispatch
minimize the total accumulated generation

\[
\begin{align*}
\text{minimize } & \theta \in \mathbb{T}^n, u \in \mathbb{R}^n \\
\text{subject to } & \\
\text{inverter power balance: } & P_i^* + u_i = P_i(\theta) \\
\text{load power balance: } & P_i^* = P_i(\theta) \\
\text{branch flow constraints: } & |\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2 \\
\text{inverter injection constraints: } & P_i(\theta) \in [0, \bar{P}_i]
\end{align*}
\]

Problem is generally non-convex and feasible only if the load is serviceable

In conventional power system operation, the economic dispatch is
• solved offline, in a centralized way, & with a model & load forecast

In an autonomously managed microgrid, the economic dispatch should be
• solved online, in a decentralized way, & without knowing a model

Objective II: decentralized dispatch optimization
Insight: droop-controlled microgrid = decentralized primal algorithm

Theorem: optimal droop
[FD, J. Simpson-Porco, & F. Bullo, ’14]
The following statements are equivalent:
(i) the economic dispatch with cost coefficients \( \alpha_i \) is strictly feasible with global minimizer \( (\theta^*, u^*) \).
(ii) \( \exists \) droop coefficients \( D_i \) such that the microgrid possesses a unique & locally exp. stable sync’d solution \( \theta \) satisfying \( P_i(\theta) \in [0, \bar{P}_i] \).
If (i) & (ii) are true, then \( \theta_i \sim \theta_i^*, u_i^* = -D_i(\omega_{\text{sync}} - \omega^*), \quad \text{and} \quad D_i \alpha_i = D_j \alpha_j \).

• similar results hold for the general constrained case
• similar results hold for the general constrained case
Secondary frequency control in power networks

**Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)

**Solution:** integral control  

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<td>• <strong>Decentralized</strong> PI control</td>
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compatible with econ. dispatch

[N. Li, L. Chen, C. Zhao, & S. Low ’13]

is globally stabilizing

[C. Zhao, E. Mallada, & FD, ’14]

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**Distributed Averaging PI (DAPI) control**

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i \dot{\theta}_i = D_i \dot{\theta}_i - \sum_{j \subseteq \text{inversors}} a_{ij} \cdot \left( \frac{\Omega_i}{D_i} - \frac{\Omega_j}{D_j} \right)$$

- no tuning & no time-scale separation: $k_i, D_i > 0$
- distributed & modular: connected comm. $\subseteq$ inverters
- recovers primary op. cond. (load sharing & opt. dispatch)

$\Rightarrow$ plug’n’play implementation

Theorem: stability of DAPI

[J. Simpson-Porco, FD, & F. Bullo, ’12]

primary droop controller works $\iff$ secondary DAPI controller works

Microgrids require **distributed** (I) secondary control strategies.
plug-and-play experiments

Plug’n’play architecture
recap of detailed signal flow (active power only)

Microgrid:
\[ P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \]
\[ Q_i = -\sum_j B_{ij} E_i E_j \]

Primary control:
mimic oscillators & polyn. symmetry

Secondary control:
diffusive averaging of injection ratios

Tertiary control:
marginal costs \( \propto 1/\text{control gains} \)

Plug’n’play architecture
similar results in the reactive case

Microgrid:
\[ P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \]
\[ Q_i = -\sum_j B_{ij} E_i E_j \]

Primary control:
mimic oscillators & polyn. symmetry

Tertiary control:
marginal costs \( \propto 1/\text{control gains} \)

Secondary control:
diffusive averaging of injection ratios
Plug’n’play architecture
experiments also work well in the coupled & lossy case

$$
\begin{align*}
\dot{P}_i &= \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j) \\
\dot{Q}_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)
\end{align*}
$$

Microgrid: physics & loadflow

Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto 1/\text{control gains}$

Secondary control: diffusive averaging of injection ratios

Experimental validation of control & opt. algorithms
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

Experimental validation of control & opt. algorithms
frequency/voltage regulation & active/reactive load sharing

**Conclusions**

1. $i \in [I_1, I_7]$ primary & tertiary control
2. $t=7s$: secondary control activated
3. $t=22s$: load $\neq 2$ unplugged
4. $t=36s$: load $\neq 2$ plugged back
Conclusions

Summary
- primary $P/\dot{\theta}$ droop control
- fair proportional load sharing & economic dispatch optimization
- distributed secondary control strategies based on averaging
- experimental validation

Further results
- reactive power control
- virtual oscillator control

Open conjecture
- solve these problems without comm

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